§ 2.1: What Is a Function

Definition of Function

A function is a rule. In order to talk about a function, we need to give it a name. We will use letters such as f, g, h, … to represent functions. For example, we may let f represent the following rule:

f is the rule "square the number"

When we write $f(3)$, we means "apply the rule f to the number 3." Applying the rule gives $f(3) = 3^2 = 9$.

Definition of Function

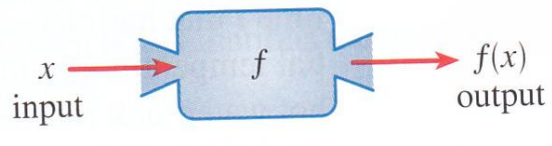
A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

We will usually consider functions for which the set A and B are sets of real numbers. The symbol $f(x)$ is read "f of x" and is called the **value of f at x.** The set A is called the **domain** of f. The **range** of f if the set of all possible values of $f(x)$ as x varies throughout the domain, in other words

$$\text{range of } f = \{f(x): x \in A\}.$$

The symbol that represents an arbitrary number in the domain of a function is called an **independent variable**. The symbol that represents a number in the range is called a **dependent variable**.

It is sometimes useful to think about functions as a machine. If x is in the domain of f, then x is taken as an input to the machine and $f(x)$ is then given as the output.



Example 1	The Squaring Function

The squaring function assigns to each real number x its square x^2. It is defined by

$$f(x) = x^2.$$

(a) Evaluate $f(3), f(-2),$ and $f(\sqrt{10})$.

(b) Find the domain and range of *f*.

(c) Draw a machine diagram for *f*.

Evaluating a Function

In the definition of a function the independent variable x plays the role of a placeholder. To evaluate *f* at a number, we substitute the number for the placeholder.

Example 2	Evaluating a Function

Let $f(x) = x^2 + 4x - 5$.

Evaluate $f(2), f(3), f\left(\frac{1}{2}\right), f(0),$ and $f(-2)$.

Example 3	A Piecewise Defined Function

An internet bookstore charges \$15 shipping for orders under \$100, but provides free shipping for orders of \$100 or more. The cost C of an order is a function of the total price x of the books purchased, given by

$$C(x) = \begin{cases} x + 15 & \text{if } x < 100 \\ x & \text{if } x \geq 100 \end{cases}$$

(a) Find $C(75), C(90), C(100), C(205)$.

(b) What do your answers in part (a) represent?

Example 4	Evaluating a Function

Let $f(x) = 2x^2 + 3x - 2$, evaluate the following

(a) $f(a)$ (b) $f(-a)$

(c) $f(a+h)$ (d) $\frac{f(a+h)-f(a)}{h}$, h ≠ 0

Example 5	The Weight of an Astronaut

If an astronaut weighs 180 lbs on the surface of the earth, then his weight when he is h miles above the earth is given by

$$w(h) = 180\left(\frac{3960}{3960+h}\right)^2$$

(a) What is his weight when he is 100 miles above the surface of the earth?

(b) Create a table of values for the function w that gives his weight at heights from 0 to 500 miles. What do you conclude from this table?

The Domain of a Function

The domain of a function is the set of all inputs for the function. The domain of a function may be stated explicitly. For example if we write

$$f(x) = x^3, \quad 0 \le x \le 10$$

then the domain is the set of all real numbers between 0 and 10 (inclusive).

If the function is given by an algebraic expression and the domain is not stated explicitly, then by convention the domain of the function is the set of all real numbers for which the expression is defined as a real number.

Example 6	Finding Domains of Functions

Find the domain of each of the following

(a) $f(x) = \frac{1}{x^2+5x}$

(b) $f(x) = \frac{1}{\sqrt{x+5}}$

(c) $f(x) = \sqrt{x^2 - 100}$

Homework

Due:_____

2 – 8 (even), 14 – 28 (even), 32, 34, 36, 44, 50, 52, 58, 62, 66, 70